

# Half-lives of $\alpha$ decay from natural nuclides and from superheavy elements

Yibin Qian<sup>a,b,\*</sup>, Zhongzhou Ren<sup>a,b,c,d,\*</sup>

<sup>a</sup>*Key Laboratory of Modern Acoustics and Department of Physics, Nanjing University, Nanjing 210093, China*

<sup>b</sup>*Joint Center of Nuclear Science and Technology, Nanjing University, Nanjing 210093, China*

<sup>c</sup>*Kavli Institute for Theoretical Physics China, Beijing 100190, China*

<sup>d</sup>*Center of Theoretical Nuclear Physics, National Laboratory of Heavy-Ion Accelerator, Lanzhou 730000, China*

---

## Abstract

Recently, experimental researches on the  $\alpha$  decay with long lifetime are one of hot topics in the contemporary nuclear physics [e.g. N. Kinoshita *et al.* (2012) [2] and J. W. Beeman *et al.* (2012) [4]]. In this study, we have systematically investigated the extremely long-lived  $\alpha$ -decaying nuclei within a generalized density-dependent cluster model involving the experimental nuclear charge radii. In detail, the important density distribution of daughter nuclei is deduced from the corresponding experimental charge radii, leading to an improved  $\alpha$ -core potential in the quantum tunneling calculation of  $\alpha$ -decay width. Besides the excellent agreement between theory and experiment, predictions on half-lives of possible candidates for natural  $\alpha$  emitters are made for future experimental detections. In addition, the recently confirmed  $\alpha$ -decay chain from  $^{294}\text{117}$  is well described, including the attractive long-lived  $\alpha$ -decaying  $^{270}\text{Db}$ , i.e., a positive step towards the “island of stability” in the superheavy mass region.

*Keywords:*

*PACS:* 23.60.+e, 21.60.Gx, 21.10.Ft, 21.10.Tg

---



---

\*Corresponding author.

*Email addresses:* [qyibin@gmail.com](mailto:qyibin@gmail.com) (Yibin Qian ), [zren@nju.edu.cn](mailto:zren@nju.edu.cn) (Zhongzhou Ren )

## 1. Introduction

Since the discovery of radioactive decays in the 1890s [1],  $\alpha$  decay has always played a quite important role in both the foundation and development of nuclear physics. In the recent experimental studies, one hot subject was to detect the naturally long-lived  $\alpha$ -decaying nuclides. A shorter  $\alpha$  decay half-life of  $^{146}\text{Sm}$  was recently measured, which appears to be quite valuable due to the significance of  $^{146}\text{Sm}$ – $^{142}\text{Nd}$  (its  $\alpha$ -decay daughter) chronology in the solar system [2]. For a long time, the naturally occurring  $^{209}\text{Bi}$  was believed to be the heaviest stable nuclide until the observation of its  $\alpha$ -decay by Marcillac *et al.* [3] and the first measurement of the partial widths by J. W. Beeman *et al.* [4]. Lead has then been supposed to be the heaviest stable element, and new experimental limits were newly proposed for the  $\alpha$  decays of Pb isotopes [5]. In fact, the detection on natural  $\alpha$  radioactivity can be dated back to 1960s [6], and it has received much more attention with the development of the facilities. Besides the above mentioned cases, much effort has been made for probing the rare  $\alpha$  activity of  $^{180}\text{W}$  [7, 8], a series of experiments were performed on the  $\alpha$  decay of natural europium [9, 10], etc. Additionally, it is exciting that the researchers have independently confirmed the existence of new element 117 [11] since the original experiment at Dubna in 2010 [12], which actually marks the official status of this new element. This newly discovered  $\alpha$  decay chain from  $^{294}117$  to a new isotope  $^{266}\text{Lr}$  even includes a hitherto longest-lived  $\alpha$ -emitters  $^{270}\text{Db}$  among heaviest elements, indicating a possible milestone towards the location of the “island of stability”. It is of great physical interest to pay attention to various long-lived  $\alpha$  emitters in nature, and the striking  $\alpha$  decay chain populating the superheavy nucleus with long lifetime.

Following the quantum explanation of  $\alpha$  decay by Gamow in 1928 [13],  $\alpha$  decay is usually considered as the tunneling process of the preformed  $\alpha$  particle through the barrier potential. With the help of phenomenological and effective  $\alpha$ -core potentials, theoretical studies [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32] have been subsequently proposed for  $\alpha$  decay calculations especially in the last two decades. Among these studies, our group provided a unified formula for half-lives of  $\alpha$  decay and cluster radioactivity [26] and a new Geiger-Nuttall relation was recently proposed for  $\alpha$  decay including the effects of the quantum numbers of  $\alpha$ -core relative motion [27]. In this Letter, we present a generalized density-dependent cluster model to depict the attractive naturally occurring  $\alpha$  emissions, in-

volving the density distributions of residual daughter nuclei based on their experimental root-mean-square (rms) charge radii. Fortunately, there are generally available experimental charge radii for these focused daughter nuclei [33]. After the total  $\alpha$ -core potential is constructed via the double-folding procedure combined with the effective M3Y-Reid-type nucleon-nucleon (NN) interaction and the standard Coulomb proton-proton interaction, the tunneling calculation is simplified as a bound state problem and a scattering state problem according to the modified two-potential theory [34]. The eigen characteristic of the bound state for the  $\alpha$  particle is determined approximately by the Wildermuth condition [35], which relates the quantum numbers of the  $\alpha$  cluster to the shell-model quantum numbers of the nucleons forming the cluster. This in fact takes into account the main requirement of the Pauli exclusion principle, and the remaining effects are absorbed into the fitting parameters of the effective  $\alpha$ -nucleus potentials.

## 2. Theoretical framework

Given the assumption that an  $\alpha$  cluster interacts with an axially symmetric deformed core nucleus, the total interaction potential of the  $\alpha$ -core system comprises of the nuclear and Coulomb potentials plus the centrifugal term,

$$V(r, \theta) = \lambda V_N(r, \theta) + V_C(r, \theta) + \frac{\hbar^2 \ell(\ell + 1)}{2\mu r^2}, \quad (1)$$

where  $\lambda$  is the renormalization factor for nuclear potential,  $\theta$  is the orientational angle of the emitted  $\alpha$  particle with respect to the symmetric axis of the daughter nucleus,  $\mu$  is the reduced mass of the  $\alpha$ -daughter system in the unit of the nucleon mass  $\mu = A_\alpha A_d / (A_\alpha + A_d)$ , and  $\ell$  is the angular momentum carried by the  $\alpha$  cluster. In the density-dependent cluster model, the nuclear and Coulomb potentials are obtained by the double-folding integral of the realistic NN interaction with the density distributions of the  $\alpha$  particle and the residual core nucleus [36, 37],

$$V_{N\text{or}C}(\mathbf{r}, \theta) = \int \int d\mathbf{r}_1 d\mathbf{r}_2 \rho_1(\mathbf{r}_1) v(\mathbf{s} = |\mathbf{r}_2 + \mathbf{r} - \mathbf{r}_1|) \rho_2(\mathbf{r}_2), \quad (2)$$

where  $v(\mathbf{s})$  denotes the widely-used M3Y NN interaction derived from the  $G$ -matrix elements of the Reid potential for the nuclear potential [36]. When

the above formula serves for the Coulomb component, the  $v(\mathbf{s})$  represents the standard Coulomb proton-proton interaction. The density distribution of the spherical  $\alpha$  particle is the standard Gaussian form given in the high-energy electron scattering experiment [37]. On the other hand, in contrast with the available information on experimental nuclear charge radii, the nuclear neutron distribution appears to be extremely ambiguous. Subsequently, the specific formula for the charge distribution can be approximately obtained based on the experimental detection, as described in the following. It is hard to analytically depict the neutron distribution from the poor knowledge of nuclear neutron radii or neutron skin thickness in nuclei [38]. Considering this, we assume that the density distribution of neutrons has the same form with that of protons in nuclei, i.e., the mass and charge density distributions of the daughter nucleus are both supposed to behave in the Fermi form,

$$\rho_2(r_2, \theta_1) = \frac{\rho_0}{1 + \exp \left[ \frac{r_2 - R(\theta_1)}{a} \right]}. \quad (3)$$

Here the half-density radius  $R(\theta_1)$  is parameterized as  $R(\theta_1) = r_0 A_d^{1/3} [1 + \beta_2 Y_{20}(\theta_1) + \beta_4 Y_{40}(\theta_1)]$ , and  $a$  is the diffuseness parameter. The  $\rho_0$  value is determined by integrating the density distribution equivalent to the mass or atomic number of the residual daughter nucleus, and the quadrupole ( $\beta_2$ ) and hexadecapole ( $\beta_4$ ) deformation parameters are taken from the theoretical values given by Möller *et al.* [39]. The  $\alpha$ -core potential can then be obtained by the double-folding integral of the effective NN interaction with the aforementioned density distributions, within the multipole expansion method (see details in Refs. [22, 37] and references therein). Given one certain angle  $\theta$ , the total potential  $V(r, \theta)$  is reduced into one dimensional case, namely  $V(r)$ . Within the two-potential approach,  $V(r)$  is then divided into two parts: the “inner” term and the “outer” term by a separation radius, and the Schrödinger equation is numerically solved in the inner potential for the bound state wave function. Because the decay energy  $Q$  is very sensitive to the half-life calculation and it cannot be predicted with sufficient accuracy for a given potential as well, we adjust the  $\lambda$  factor to the experimental  $Q$  value for each decay. Meanwhile, to reflect the Pauli exclusion principle, the quantum number  $n$  of the bound solution (i.e., the number of internal nodes) is chosen by the Wildermuth condition [35],

$$G = 2n + \ell = \sum_{i=1}^4 g_i. \quad (4)$$

In this expression,  $g_i$  are the corresponding oscillator quantum numbers of the ingredient nucleons in the  $\alpha$  cluster, whose values are restricted to ensure the  $\alpha$  cluster completely outside the shell occupied by the core nucleus. Here we take  $g_i = 4$  for nucleons with  $50 \leq Z, N \leq 82$ ,  $g_i = 5$  for nucleons with  $82 < Z, N \leq 126$ , and  $g_i = 6$  for nucleons beyond the  $N = 126$  neutron shell closure. Moreover, a zero-range term for the single-nucleon exchange is introduced in the M3Y NN interaction to guarantee the antisymmetrization of identical nucleons in the  $\alpha$  cluster and in the core nucleus [37]. Subsequently, one can use the wave function to obtain the  $\alpha$  decay width  $\Gamma(\theta)$  for the given angle, as described in previous studies [30, 31]. By averaging the width in various directions [16, 17, 22], the final decay width is given by

$$\Gamma = \int_0^{\pi/2} \Gamma(\theta) \sin(\theta) d\theta. \quad (5)$$

Previously, the parameters  $r_0$  and  $a$  in the density distribution are suggested at  $r_0 = 1.07$  fm and  $a = 0.54$  fm from the nuclear textbook [40], which could lead to the calculated decay width. In the present Letter, we make use of the corresponding experimental nuclear charge radii to determine the related parameters to pursue a better description of the naturally occurring  $\alpha$  activities with long half-lives. In detail, the  $\alpha$  particle in the decay process is usually considered to be formed in nuclear surface, which seems to be directly related with the half-density radius, namely  $r_0$  factor. Importantly, besides the intuitive knowledge, we found that the final decay width is more sensitive to the quantity  $r_0$  as compared to the diffuseness parameter  $a$  [38]. On the other hand, the focused natural  $\alpha$  emitters are generally in the medium mass region of nuclide chart. While the diffuseness value ( $a = 0.54$  fm) is suitable for the  $\alpha$  decay studies in heavy nuclei [22, 38], the  $a$  values in the density distribution should be relatively less ones for medium nuclei. Based on these facts, the diffuseness  $a$  value is fixed at the following constants:  $a = 0.54$  fm for heavy nuclei with  $N > 126$ , and  $a = 0.52$  fm for medium nuclei with  $N \leq 126$ .  $r_0$  is then considered as the representation factor of the rms nuclear charge radii. The  $r_0$  value of  $\rho_2$  for daughter nuclei can be conveniently obtained from the experimental charge radii by the relationship

$$R \equiv \sqrt{\langle r^2 \rangle} = \left[ \frac{\int \rho_2(r, \theta_1) r^4 \sin \theta_1 dr d\theta_1}{\int \rho_2(r, \theta_1) r^2 \sin \theta_1 dr d\theta_1} \right]^{1/2}. \quad (6)$$

After the decay width is proceeded through the above sequential proce-

ture, the  $\alpha$  decay half-life is ultimately related as

$$T_{1/2} = \frac{\hbar \ln 2}{P_\alpha \Gamma}, \quad (7)$$

where the  $\alpha$ -preformation factor  $P_\alpha$  inscribes the preformation probability of an  $\alpha$  cluster in the parent nucleus. Its value can, in principle, be evaluated from the overlap between the actual wave function of the parent nucleus and that of the decaying state depicting the  $\alpha$  cluster coupled to the residual daughter nucleus. However, it is in fact extremely difficult to achieve these wave functions due to the complexity of both the nuclear potential and the nuclear many-body problem. According to the experimental analysis, the preformation factor should vary smoothly in the open-shell region and has a value less than unity [41]. Considering this, the  $\alpha$ -preformation factor is taken as the same constant for one kind of nuclei, to keep the minimum of free parameters in the model as well. Through a least square fit to the experimental half-lives of those long-lived  $\alpha$  emitters, the  $P_\alpha$  values are chosen as:  $P_\alpha^{e-e} = 0.42$  for even-even nuclei and  $P_\alpha^{odd-A} = 0.15$  for odd- $A$  nuclei. This is consistent with the Buck's model [14], and these values are close to the microscopic calculation of the typical nucleus  $^{212}\text{Po}$  [15]. There is no doubt that the experimental  $\alpha$  decay half-lives should be better reproduced if the preformation factor is considered as a variable along with different parent nuclei instead of a constant. Several detections have been performed for this subject, especially for the closed-shell nuclei [28, 29, 30, 31, 42]. This deserves further investigation.

### 3. Numerical results and discussions

We initially pay main attention to the long-lived  $\alpha$ -decaying nuclides in nature, within a generalized density-dependent cluster model as described above. Table 1 presents our calculated results for the  $\alpha$  decay properties of these focused emitters, which generally decay from ground states to ground states as listed in the first column. The next two columns list the experimental decay energies  $Q$  and half-lives, which are taken from the AME2012 [43], the NNDC [44] databases, and the newly detected data within improved accuracy [2, 3, 4, 5, 10]. The fourth and fifth columns denote the experimental charge radii of daughter nuclei [33] and the extracted  $r_0$  values in the density distribution [Eq. (3)], respectively. Additionally, the renormalization factor  $\lambda$ , namely another important quantity, is determined in the aforementioned

calculation process and actually varies in a small range of 0.613-0.707. The present calculated results are given in the sixth column. In detail, these  $\alpha$  decays usually choose the favored ones with  $\ell = 0$  on the basis of the available experimental assignments, except for  $^{151}\text{Eu}$  and  $^{209}\text{Bi}$ . According to the new discovery [9, 10], the spin and parity of  $^{151}\text{Eu}$  and  $^{147}\text{Pm}$  are respectively assigned as  $5/2^+$  and  $7/2^+$ , leading to the minimum  $\ell = 2$  following the spin-parity selection rule. It should also be noted that the nuclear charge radius of  $^{147}\text{Pm}$  is taken from the estimation and systematics of the isotopic chain due to its absence in experiment [33]. For the  $\alpha$  decay of  $^{209}\text{Bi}$ , the angular momentum transferred by the cluster should be  $\ell = 5$ , resulting from the decay scheme  $9/2^- \rightarrow 1/2^+$  [3, 4]. Simultaneously, the transition from  $^{209}\text{Bi}$  is strongly effected by the neutron closed-shell ( $N = 126$ ) and its  $P_\alpha$  value has to be exclusively chosen as the same one proposed in our previous studies on exotic  $\alpha$  decays [30]. Moreover, the last two columns in Table 1 respectively list the results obtained by the united model for  $\alpha$  decay and  $\alpha$  capture UMADAC (Ref. [17] and codes therein), and the analytic expressions for  $\alpha$  decay half-lives for full set of nuclei [19], to preform the comparison of the present approach with other ones.

Generally, it is found that our calculated half-lives well agree with the experimental data within a mean factor of 1.5, and are comparable to the values given by some other models. Especially, our calculations are very close to experiments performed for the important clock  $^{146}\text{Sm}$  in the solar system, the very newly detected  $^{151}\text{Eu}$ , the unexpected  $\alpha$ -emitter  $^{209}\text{Bi}$ , etc. This may imply the significance of considering the sensitive quantity  $r_0$  from the nuclear charge radii of daughter nuclei for computing  $\alpha$  decay half-lives. Encouraged by this, we have also provided predictions on  $\alpha$  decay half-lives for candidates of naturally long-lived  $\alpha$  emitters. The rare  $\alpha$  radioactivities of  $^{149}\text{Sm}$ ,  $^{174,176,178}\text{Hf}$ ,  $^{184}\text{Os}$  and  $^{192}\text{Pt}$  are strongly suggested for future experimental researches, in view of their appropriate predicted half-lives. For example, there is still the uncertainty for the decay energy of  $^{174}\text{Hf}$  [43, 44], and its experiment can serve for the isotopes as well.

As an additional test, we have performed an investigation (listed in Table 2) on the decay properties of the  $\alpha$  decay chain originated from  $^{294}117$ , newly observed in the important experiment confirming the existence of the new element. The  $\alpha$  decay chains from this newly discovered element 117 have in fact received a lot of attention in theoretical studies (see Refs. [19, 20, 21, 32] and references therein) based on different models such as the shell model [20] and the Coulomb and proximity potential model for de-

formed nuclei [21], in which the calculated results are generally consistent with each other. One can see that the new experimental data [11] exactly provide an opportunity to further check the validity of the theoretical model. Unfortunately, there is little knowledge of the level scheme and unavailable information on the nuclear charge radii of nuclei in the superheavy mass region. Hence we assume that the  $\alpha$  transitions are favored (namely,  $\ell = 0$ ) for these superheavy nuclei, and the key parameter  $r_0$  and the diffuseness  $a$  in their density-distributions are still fixed at the standard values proposed in the textbook [22, 40], as mentioned before. The  $\alpha$ -preformation factor is then taken as the same choice with our previous studies [31, 32] for this case. Furthermore, there are two possible values of decay energies for  $^{282}\text{Rg}$  in the new measurement, and we have chosen the identical one with the previous experiment [12].

With these above in mind, we have calculated the  $\alpha$  decay half-lives by using the experimental decay energies [11]. Due to the rare events of experiments, there are slightly large error bars of decay energies for the  $\alpha$  decay chain from  $^{294}117$ . On one hand, these experimental  $Q$  values are compatible with those in the AME2012 tables [43]. Some deviations of them may result from the reason that the mass tables offer the decay energies between ground states while the measurements are proceeded for the transitions from or to low exciting states. On the other hand, the calculated half-live depends strongly on the decay energy  $Q$ . There are usually discrepancies of  $Q$  values in various experimental works and theoretical mass tables especially for superheavy nuclei, which correspondingly bring different half-lives in extensive theoretical studies. Despite this, the present calculation and conclusion are not affected. As one can see from Table 2, our calculated half-lives, located in a certain range, slightly underestimate the corresponding experimental values. This appears to be reasonable because of the possible disregard for unfavored cases ( $\ell \neq 0$ ), which could increase the calculated half-lives. For the abnormal discrepancy in  $^{286}113$ , we may need the enough experimental recognition of nuclear deformation, energy level and nuclear radius to improve the calculated result. However, the consistency of the calculation with the measurement is well reached for these isotopes including the long-lived  $\alpha$ -decaying  $^{270}\text{Db}$ , which may be an important nucleus towards the “island of stability” in the heaviest regime. The following work about the predictions on the attractive “island of stability” is being in the process.



#### 4. Summary

In summary, we have developed the density-dependent cluster model to carefully investigate the naturally  $\alpha$ -decaying nuclei with long lifetimes, especially for the newly discovered nuclides and the important  $\alpha$ -emitters with improved measurements. The sensitive parameter in the density distribution of daughter nuclei in the decay process is obtained from the experimental nuclear charge radius, in order to pursue an enhanced description of  $\alpha$ -core potential via the double-folding method. Our sequential calculations give the theoretical results of  $\alpha$  decay half-lives, excellently agreeing with the experimental data and being comparable with other theoretical values. As well, we have made a series of predictions on long half-lives for possible  $\alpha$ -decaying nuclei in nature, to be strongly suggested for future detections. The reconfirming  $\alpha$  decay chain from the new element 117 have also been focused on, to actually further check the validity of our model to some extent.

#### Acknowledgments

We thank Dongdong Ni and Renli Xu for carefully reading the manuscript. This work is supported by the National Natural Science Foundation of China (Grants No. 11035001, No. 10975072, No. 10735010, No. 11375086, and No. 11120101005), by the 973 National Major State Basic Research and Development of China (Grants No. 2010CB327803 and No. 2013CB834400), by CAS Knowledge Innovation Project No. KJCX2-SW-N02, by Research Fund of Doctoral Point (RFDP), Grant No. 20100091110028, and by the Project Funded by the Priority Academic Programme Development of JiangSu Higher Education Institutions (PAPD).

#### References

- [1] H. Becquerel, CR Acad. Sci 122 (1896) 501.
- [2] N. Kinoshita *et al.*, Science 335 (2012) 1614.
- [3] P. D. Marcillac, N. Coron, G. Dambier, J. Leblanc, and J. Moalic, Nature (London) 422 (2003) 876.
- [4] J. W. Beeman *et al.*, Phys. Rev. Lett. 108 (2012) 062501.

- [5] J. W. Beeman *et al.*, Eur. Phys. J. A 49 (2013) 50.
- [6] R. D. Macfarlane, T. P. Kohman, Phys. Rev. 121 (1961) 1758.
- [7] F. A. Danevich *et al.*, Phys. Rev. C 67 (2003) 014310.
- [8] C. Cozzini *et al.*, Phys. Rev. C 70 (2004) 064606.
- [9] P. Belli *et al.*, Nucl. Phys. A 789 (2007) 15.
- [10] N. Casali *et al.*, J. Phys. G: Nucl. Part. Phys. 41 (2014) 075101.
- [11] J. Khuyagbaatar *et al.*, Phys. Rev. Lett. 112 (2014) 172501.
- [12] Yu. Ts. Oganessian *et al.*, Phys. Rev. Lett. 104 (2010) 142502;  
Yu. Ts. Oganessian *et al.*, Phys. Rev. C 83 (2011) 054315.
- [13] G. Gamow, Z. Phys. 51 (1928) 204.
- [14] B. Buck, A. C. Merchant, S. M. Perez, Phys. Rev. Lett. 65 (1990) 2975;  
B. Buck, A. C. Merchant, S. M. Perez, At. Data Nucl. Data Tables 54 (1993) 53.
- [15] R. G. Lovas, R. J. Liotta, A. Insolia, K. Varga, D. S. Delion, Phys. Rep. 294 (1998) 265.
- [16] V. Yu. Denisov, H. Ikezoe, Phys. Rev. C 72 (2005) 064613.
- [17] KINR UMADAC Code: <http://www.nndc.bnl.gov/codes/UMADAC/>;  
V. Yu. Denisov, A. A. Khudenko, At. Data Nucl. Data Tables 95 (2009) 815; At. Data Nucl. Data Tables 97 (2011) 187 (E).
- [18] G. Royer, J. Phys. G: Nucl. Part. Phys. 26 (2000) 1149.
- [19] G. Royer, Nucl. Phys. A 848 (2010) 279.
- [20] I. Silisteanu, A. I. Budaca, At. Data Nucl. Data Tables 98 (2012) 1096.
- [21] K. P. Santhosh, B. Priyanka, M. S. Unnikrishnan, Phys. Rev. C 85 (2012) 034604.
- [22] Chang Xu, Zhongzhou Ren, Nucl. Phys. A 760 (2005) 303;  
Chang Xu, Zhongzhou Ren, Phys. Rev. C 73 (2006) 041301(R).

- [23] P. Mohr, Phys. Rev. C 73 (2006) 031301(R).
- [24] D. N. Poenaru, I. H. Plonski, W. Greiner, Phys. Rev. C 74 (2006) 014312.
- [25] Zhongzhou Ren, Chang Xu, Zaijun Wang, Phys. Rev. C 70 (2004) 034304.
- [26] Dongdong Ni, Zhongzhou Ren, Tiekuan Dong, Chang Xu, Phys. Rev. C 78 (2008) 044310.
- [27] Yuejiao Ren, Zhongzhou Ren, Phys. Rev. C 85 (2012) 044608.
- [28] M. Ismail, A. Adel, Phys. Rev. C 89 (2014) 034617.
- [29] W. M. Seif, J. Phys. G: Nucl. Part. phys. 40 (2013) 105102.
- [30] Yibin Qian, Zhongzhou Ren, Nucl. Phys. A 852 (2011) 82.
- [31] Yibin Qian, Zhongzhou Ren, Dongdong Ni, Nucl. Phys. A 866 (2011) 1.
- [32] Yibin Qian, Zhongzhou Ren, Phys. Rev. C 84 (2011) 064307.
- [33] I. Angeli, K. P. Marinova, At. Data Nucl. Data Tables 99 (2013) 69.
- [34] S. A. Gurvitz, P. B. Semmes, W. Nazarewicz, T. Vertse, Phys. Rev. A 69 (2004) 042705.
- [35] K. Wildermuth, Y. C. Tang, *A Unified Theory of the Nucleus* Academic Press, New York, 1997.
- [36] G. F. Bertsch, J. Borysowicz, H. Mcmanus, W. G. Love, Nucl. Phys. A 284 (1977) 399.
- [37] G. R. Satchler, W. G. Love, Phys. Rep. 55 (1979) 183.
- [38] Dongdong Ni, Zhongzhou Ren, Tiekuan Dong, Yibin Qian, Phys. Rev. C 87 (2013) 024310.
- [39] P. Möller, J. R. Nix, W. D. Myers, W. J. Swiatecki, At. Data Nucl. Data Tables 59 (1995) 185.
- [40] J. D. Walecka, *Theoretical Nuclear Physics and Subnuclear Physics* Oxford University, Oxford, 1995, p. 11.

- [41] P. E. Hodgson, E. Běták, Phys. Rep. 374 (2003) 1.
- [42] Yibin Qian, Zhongzhou Ren, Sci. China-Phys. Mech. Astron. 56 (2013) 1520.
- [43] G. Audi, F. G. Kondev, M. Wang, B. Pfeiffer, X. Sun, J. Blachot, M. MacCormick, Chin. Phys. C 36 (2012) 1157.
- [44] National Nuclear Data Center, Brookhaven National Laboratory [<http://www.nndc.bnl.gov>].

Table 1: Comparison of calculated  $\alpha$  decay half-lives based on the corresponding measured charge radii with available experimental values and other theoretical results for long-lived  $\alpha$ -decaying nuclei ( $T_{1/2}$  in years), including the improved or new data about  $^{146}\text{Sm}$ ,  $^{151}\text{Eu}$  and so on. The last two columns denote the calculations within the UMADAC model [17] and the analytic formulas given by Royer [19]. Predicted half-lives for hitherto undetected  $\alpha$  emitters in nature are provided as well.

| Decay   | $Q(\text{MeV})$ | $R^{\text{expt}}(\text{fm})$ | $r_0$ | $T_{1/2}^{\text{expt}}$            | $T_{1/2}^{\text{calc}}$ | $T_{1/2}^{\text{UMADAC}}$ | $T_{1/2}^{\text{form}}$ |
|---|-----------------|------------------------------|-------|------------------------------------|-------------------------|---------------------------|-------------------------|
| $^{144}\text{Nd} \rightarrow ^{140}\text{Ce}$ | 1.905           | 4.88                         | 1.113 | $2.29 \pm 0.16 \times 10^{15}$     | $3.14 \times 10^{15}$   | $1.36 \times 10^{16}$     | $3.32 \times 10^{15}$   |
| $^{146}\text{Sm} \rightarrow ^{142}\text{Nd}$ | 2.529           | 4.91                         | 1.118 | $6.8 \pm 0.7 \times 10^7$          | $6.9 \times 10^7$       | $2.0 \times 10^8$         | $1.0 \times 10^8$       |
| $^{147}\text{Sm} \rightarrow ^{143}\text{Nd}$ | 2.311           | 4.93                         | 1.123 | $1.06 \pm 0.02 \times 10^{11}$     | $1.32 \times 10^{11}$   | $1.00 \times 10^{12}$     | $2.51 \times 10^{11}$   |
| $^{148}\text{Sm} \rightarrow ^{144}\text{Nd}$ | 1.986           | 4.94                         | 1.120 | $7.00 \pm 2.00 \times 10^{15}$     | $7.98 \times 10^{15}$   | $4.48 \times 10^{16}$     | $8.36 \times 10^{15}$   |
| $^{151}\text{Eu} \rightarrow ^{147}\text{Pm}$ | 1.9489          | 4.99                         | 1.075 | $4.62 \pm 1.63 \times 10^{18}$     | $3.71 \times 10^{18}$   | $5.95 \times 10^{18}$     | $1.28 \times 10^{19}$   |
| $^{152}\text{Gd} \rightarrow ^{148}\text{Sm}$ | 2.205           | 5.00                         | 1.075 | $1.08 \pm 0.08 \times 10^{14}$     | $1.99 \times 10^{14}$   | $5.02 \times 10^{14}$     | $1.18 \times 10^{14}$   |
| $^{180}\text{W} \rightarrow ^{176}\text{Hf}$  | 2.516           | 5.33                         | 1.082 | $1.1^{+0.9}_{-0.5} \times 10^{18}$ | $7.18 \times 10^{17}$   | $2.79 \times 10^{18}$     | $2.95 \times 10^{17}$   |
| $^{186}\text{Os} \rightarrow ^{182}\text{W}$  | 2.822           | 5.36                         | 1.085 | $2.0 \pm 1.1 \times 10^{15}$       | $1.34 \times 10^{15}$   | $4.59 \times 10^{15}$     | $5.99 \times 10^{14}$   |
| $^{190}\text{Pt} \rightarrow ^{186}\text{Os}$ | 3.243           | 5.39                         | 1.097 | $6.50 \pm 0.30 \times 10^{11}$     | $3.51 \times 10^{11}$   | $1.31 \times 10^{12}$     | $2.04 \times 10^{11}$   |
| $^{209}\text{Bi} \rightarrow ^{205}\text{Tl}$ | 3.137           | 5.48                         | 1.129 | $2.03 \pm 0.08 \times 10^{19}$     | $1.78 \times 10^{19}$   | $2.96 \times 10^{20}$     | $3.21 \times 10^{19}$   |
| $^{244}\text{Pu} \rightarrow ^{240}\text{U}$  | 4.666           | 5.87                         | 1.072 | $1.007 \pm 0.004 \times 10^8$      | $1.498 \times 10^8$     | $4.459 \times 10^8$       | $1.295 \times 10^8$     |
| $^{142}\text{Ce} \rightarrow ^{138}\text{Ba}$ | 1.310           | 4.84                         | 1.108 | $> 5 \times 10^{16}$               | $4.02 \times 10^{27}$   | $3.43 \times 10^{28}$     | $2.37 \times 10^{27}$   |
| $^{145}\text{Nd} \rightarrow ^{141}\text{Ce}$ | 1.578           | 4.93                         | 1.129 |                                    | $6.00 \times 10^{22}$   | $1.07 \times 10^{24}$     | $1.60 \times 10^{23}$   |
| $^{149}\text{Sm} \rightarrow ^{145}\text{Nd}$ | 1.870           | 4.95                         | 1.076 | $> 2 \times 10^{15}$               | $5.84 \times 10^{18}$   | $3.06 \times 10^{19}$     | $6.52 \times 10^{18}$   |
| $^{156}\text{Dy} \rightarrow ^{152}\text{Gd}$ | 1.758           | 5.08                         | 1.072 | $> 1.0 \times 10^{15}$             | $5.98 \times 10^{24}$   | $2.81 \times 10^{25}$     | $1.88 \times 10^{24}$   |
| $^{162}\text{Er} \rightarrow ^{158}\text{Dy}$ | 1.645           | 5.18                         | 1.061 | $> 1.4 \times 10^{14}$             | $4.56 \times 10^{29}$   | $1.80 \times 10^{30}$     | $9.23 \times 10^{28}$   |
| $^{164}\text{Er} \rightarrow ^{160}\text{Dy}$ | 1.304           | 5.20                         | 1.059 |                                    | $2.22 \times 10^{40}$   | $1.69 \times 10^{41}$     | $2.05 \times 10^{39}$   |
| $^{168}\text{Yb} \rightarrow ^{164}\text{Er}$ | 1.950           | 5.24                         | 1.069 | $> 1.3 \times 10^{14}$             | $2.92 \times 10^{24}$   | $1.13 \times 10^{25}$     | $8.05 \times 10^{23}$   |
| $^{174}\text{Hf} \rightarrow ^{170}\text{Yb}$ | 2.559           | 5.29                         | 1.070 | $2.00 \pm 0.40 \times 10^{15}$     | $4.43 \times 10^{15}$   | $1.07 \times 10^{16}$     | $2.02 \times 10^{15}$   |
| $^{176}\text{Hf} \rightarrow ^{172}\text{Yb}$ | 2.258           | 5.30                         | 1.073 |                                    | $2.25 \times 10^{20}$   | $7.76 \times 10^{20}$     | $7.67 \times 10^{19}$   |
| $^{178}\text{Hf} \rightarrow ^{174}\text{Yb}$ | 2.083           | 5.31                         | 1.078 |                                    | $3.34 \times 10^{23}$   | $1.59 \times 10^{24}$     | $9.54 \times 10^{22}$   |
| $^{182}\text{W} \rightarrow ^{178}\text{Hf}$  | 1.774           | 5.34                         | 1.083 |                                    | $2.66 \times 10^{32}$   | $2.82 \times 10^{33}$     | $3.97 \times 10^{31}$   |
| $^{184}\text{Os} \rightarrow ^{180}\text{W}$  | 2.957           | 5.35                         | 1.085 |                                    | $2.85 \times 10^{13}$   | $9.46 \times 10^{13}$     | $1.44 \times 10^{13}$   |
| $^{188}\text{Os} \rightarrow ^{184}\text{W}$  | 2.143           | 5.37                         | 1.092 |                                    | $1.02 \times 10^{26}$   | $8.17 \times 10^{26}$     | $2.27 \times 10^{25}$   |
| $^{192}\text{Pt} \rightarrow ^{188}\text{Os}$ | 2.418           | 5.40                         | 1.103 | $> 6 \times 10^{16}$               | $5.09 \times 10^{22}$   | $4.72 \times 10^{23}$     | $1.46 \times 10^{22}$   |
| $^{196}\text{Hg} \rightarrow ^{192}\text{Pt}$ | 2.041           | 5.42                         | 1.152 | $> 2.5 \times 10^{18}$             | $3.51 \times 10^{31}$   | $1.16 \times 10^{33}$     | $1.14 \times 10^{31}$   |
| $^{204}\text{Pb} \rightarrow ^{200}\text{Hg}$ | 1.9695          | 5.46                         | 1.143 | $> 1.4 \times 10^{20}$             | $1.70 \times 10^{35}$   | $7.94 \times 10^{36}$     | $2.82 \times 10^{34}$   |

Table 2: Calculated  $\alpha$  decay half-lives in the decay chain from the new nuclide  $^{294}\text{117}$ , compared with the experimental values and other theoretical model calculations (the UMADAC model [17] and the analytical formulas [19]). The measured data, i.e., the decay energies  $Q$  and the half-lives  $T_{1/2}^{\text{expt}}$ , are obtained from the very recent experiment [11].

| Nucleus            | $Q(\text{MeV})$ | $T_{1/2}^{\text{expt}}$ | $T_{1/2}^{\text{calc}}$ | $T_{1/2}^{\text{UMADAC}}$ | $T_{1/2}^{\text{form}}$ |
|--------------------|-----------------|-------------------------|-------------------------|---------------------------|-------------------------|
| $^{294}\text{117}$ | 11.20(4)        | $51_{-20}^{+94}$ ms     | 22-35 ms                | 119-190 ms                | 33-53 ms                |
| $^{290}\text{115}$ | 10.45(4)        | $1.3_{-0.5}^{+2.3}$ s   | 0.4-0.7 s               | 3.3-5.6 s                 | 0.7-1.1 s               |
| $^{286}\text{113}$ | 9.4(3)          | $2.9_{-1.1}^{+5.3}$ s   | 12.0-943.3 s            | 102.5-9468.6 s            | 19.9-1567.7 s           |
| $^{282}\text{Rg}$  | 9.18(3)         | $3.1_{-1.2}^{+5.7}$ min | 1.5-2.3 min             | 11.4-17.9 min             | 2.7-4.2 min             |
| $^{278}\text{Mt}$  | 9.59(3)         | $3.6_{-1.4}^{+6.5}$ s   | 0.8-1.3 s               | 4.3-6.5 s                 | 1.9-2.8 s               |
| $^{274}\text{Bh}$  | 8.97(3)         | $30_{-12}^{+54}$ s      | 12-18 s                 | 59-93 s                   | 27-43 s                 |
| $^{270}\text{Db}$  | 8.02(3)         | $1.0_{-0.4}^{+1.9}$ h   | 1.0-1.6 h               | 5.7-9.6 h                 | 2.4-3.9 h               |